Resonated Libration of Tethered Subsatellite by Atmospheric Density Variation

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Nomenclature

A = projected area of subsatellite normal to its velocity
C_D = aerodynamic drag coefficient
h = altitude of orbit
ℓ = length of tether
m = mass of subsatellite
R = radius of orbit
R_E = radius of the Earth
θ = librational angle between local vertical and tether line, see Fig. 3
ρ = atmospheric density
Ψ = true anomaly, see Fig. 2
ω = orbital angular velocity

Subscripts

0 = steady-state solution
1 = condition of day
2 = condition of night

Introduction

A LARGE number of researchers have studied dynamics and/or control of tethered subsatellite systems (TSS).[1] Concerning the dynamics during the stationkeeping phase in a low-altitude Earth orbit, Bainum et al.[2] showed that the instability of out-of-plane motion with high inclination orbit is caused by a forced resonance arising from the rotating atmosphere. On the other hand, Onoda and Watanabe[3] reported that the in-plane-motion instability is caused by the atmospheric density gradient. To the best of the author's knowledge, the atmospheric density in the literature has been regarded as a function of altitude only. The atmospheric density is, however, different during the day from that during the night.[4] This Note investigates the effect of this atmospheric density variation on the in-plane librational motion during the stationkeeping phase.

Equations of Motion

To clarify the effect of the atmospheric density variation, the mathematical model of the present system is simplified as much as possible by making the following assumptions.

1) The Earth is spherical. The orbit is circular, and the orbital plane coincides with the ecliptic plane.

2) The subsatellite is spherical and can be regarded as a mass point.

3) The mass of the subsatellite is sufficiently small compared to that of the main body. Hence, the center of mass of the present TSS can be regarded to coincide with the center of mass of the main body.

4) The tether is massless and rigid. Its length is constant.

5) The atmosphere has no motion with respect to the inertial space. Its density is a function of altitude and true anomaly. Figure 1 shows the variation of the atmospheric density with respect to the altitude.[4] Curves are drawn for day and night and for periods of high and low solar activity. Figure 2 shows the orbital plane, where Ψ denotes true anomaly measured from the local vertical parallel to the sunbeam. In the present Note, night is defined as the part of the orbit that is in the shadow of the Earth ($\Psi_1 \leqq \Psi < \Psi_1 + \Psi_2$), whereas day is defined as the part not in the shadow of the Earth ($-\Psi_1 \leqq \Psi < \Psi_1$).

6) Aerodynamic drag is proportional to the atmospheric density, square of the subsatellite's velocity relative to the atmosphere, and projected area of the subsatellite normal to its velocity. The aerodynamic drag along the orbit discontinuously changes at $\Psi = \Psi_1$ and $\Psi_1 + \Psi_2$ because the atmospheric density changes from daytime to nighttime value at $\Psi = \Psi_1$ and from nighttime to daytime value at $\Psi = \Psi_1 + \Psi_2$.

7) The external forces affecting the motion are only the gravitational force and the aerodynamic drag.

Figure 3 shows a schematic representation of the present TSS. The equations of librational motion of the subsatellite then can be written as follows:

$$\ddot{\theta} + 3\omega^2 \sin\theta\cos\theta = C_1\omega^2\cos\theta \qquad (-\Psi_1 \leqq \Psi < \Psi_1)$$
$$\ddot{\theta} + 3\omega^2 \sin\theta\cos\theta = C_2\omega^2\cos\theta \qquad (\Psi_1 \leqq \Psi < \Psi_1 + \Psi_2) \tag{1}$$

where

$$C_i = \frac{1}{2}\left(\frac{C_D A \rho_i R^2}{m\ell}\right) \qquad (i = 1, 2) \tag{2}$$

and

$$R = R_E + h \tag{3}$$

Linearization of the Equations of Motion

Steady-state solution of Eq. (1) satisfies the following equation:

$$\sin\theta_{0i} = C_i/3 \tag{4}$$

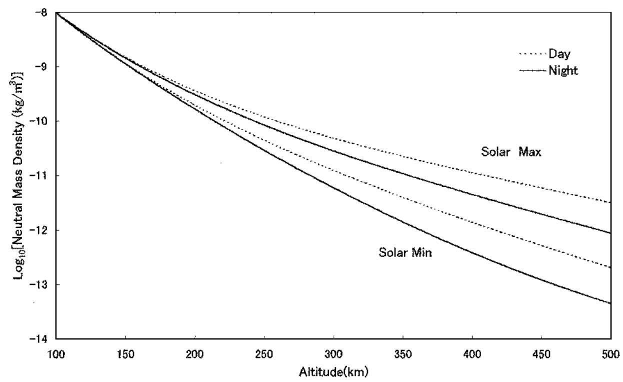

Fig. 1 Variation of atmospheric density with altitude.

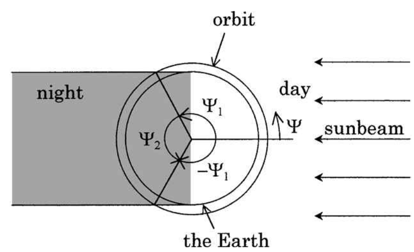

Fig. 2 Day/night geometry in orbital plane.

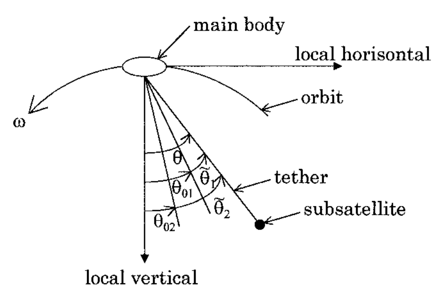

Fig. 3 Schematic representation of TSS.

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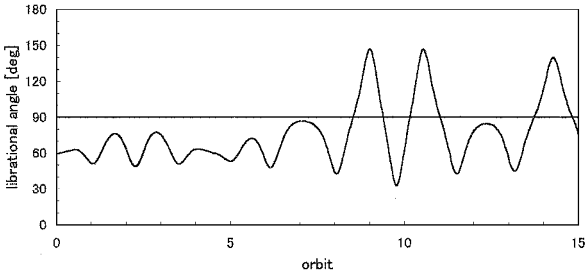

Fig. 4 Variation of librational angle.

The equation of infinitesimal additional motion around the steady-state solution can be obtained as follows:

$$\ddot{\tilde{\theta}}_i + (p_i\omega)^2\tilde{\theta}_i = 0 \tag{5}$$

where

$$p_i = \sqrt{3 - \left(C_i^2/3\right)} \tag{6}$$

and

$$\theta = \theta_{0i} + \tilde{\theta}_i \tag{7}$$

as shown in Fig. 3.

Forced Resonance Due to Atmospheric Density Variation

It is easy to see that forced resonance may occur when the angular velocity of the subsatellite libration is equal to a multiple of the orbital angular velocity as follows:

$$p_i\omega = n\omega \qquad (n = 1, 2, 3, \ldots) \tag{8}$$

Substituting Eqs. (2), (3), and (6) into Eq. (8), one can arrive at an equation describing the condition of the forced resonance due to the atmospheric density variation as

$$\frac{C_D A}{m\ell} = \frac{2}{\rho_i(R_E + h)^2}\sqrt{9 - 3n^2} \tag{9}$$

Equation (9) has real solution only in the case where $n = 1$, that is, the tethered subsatellite librates once per orbit. Thus, Eq. (9) yields

$$\frac{C_D A}{m\ell} = \frac{2\sqrt{6}}{\rho_i(R_E + h)^2} \tag{10}$$

where the left-hand side of Eq. (10) consists of the parameters dependent on the form of the TSS; on the other hand, the right-hand side of Eq. (10) is a function of the altitude h.

Numerical Example

Suppose R_E, h, and ρ_2 to be 6378 km, 160 km, and 1.04×10^{-9} kg/m^3, respectively; then the value of $2\sqrt{6}/[\rho_2(R_E+h)^2]$ is obtained to be 1.10×10^{-4}. If parameters C_D, A, m, and ℓ are selected as 2.2, 1.0 m^2, 20 kg, and 1000 m, respectively, then the value of $C_D A/m\ell$ is also equal to 1.10×10^{-4}. Now, Eq. (10) is satisfied, and the tethered subsatellite may be forced to resonate by the atmospheric density variation.

Figure 4 shows the time history of the libration θ by using Eq. (1) with the initial conditions: $\Psi = -\Psi_1$, $\theta = \theta_{01}$, and $\dot{\theta} = 0$. It can be seen that the libration of the tethered subsatellite diverges and crosses the local horizontal in nine orbits.

Conclusions

It is concluded that the librational motion of the tethered subsatellite during the stationkeeping phase is forced to resonate by the atmospheric density variation. The numerical result shows that the libration diverges and crosses the local horizontal in nine orbits when the angular velocity of the TSS's libration is equal to the orbital angular velocity. This result also indicates a possibility that the librational motion during the deployment and retrieval phases may be induced to diverge by the external perturbation due to the atmospheric density variation.

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Worst-Case Control for Discrete-Time Uncertain Nonlinear Systems

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Introduction

ONE of the important requirements for a control system is the so-called robustness. During the last decade, robust control of uncertain dynamic systems has been extensively studied. Considerable attention has been devoted to the problems of robust stabilization and robust performance of uncertain dynamic systems in both the continuous and discrete-time contexts.[1,2]

We extend the control design methodology proposed in Ref. 3 to handle the problem of robust control of a class of uncertain discrete-time systems. The methodology is extended in two ways. The first idea is to allow parametric uncertainty in the controlled output equation. The second aspect is that the nonlinear uncertainty contains system state, control input, and disturbance input variables all together. The class of uncertain systems is described by a state-space model with linear nominal parts and nonlinear time-varying norm-bounded parameter uncertainty in the state and output equations. Attention is focused on the design of linear state feedback controllers. We address the problem of robust H_∞ control in which both robust stability and a prescribed H_∞ performance are required to be achieved irrespective of the uncertainties. Our results show that the described problem can be converted to a robust H_∞ control for a related discrete-time system with time-varying norm-bounded linear uncertainty. Therefore, the Riccati inequality approach[4] can be used to obtain a solution to the problem of robust H_∞ control of nonlinear uncertain systems.

Robust Control Result

Consider the following uncertain system:

$$(\Sigma): \qquad x_{k+1} = Ax_k + B_1 w_k + B_2 u_k + \Delta_1(x_k, w_k, u_k)$$

$$x_0 = 0 \tag{1}$$

$$z_k = Cx_k + D_1 w_k + D_2 u_k + \Delta_2(x_k, w_k, u_k) \tag{2}$$

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